I J R B A T, Issue (XI) Vol (II) May 2023 : 295-305

A Double-Blind Peer Reviewed & Refereed Journal

OPEN ACCESS

e-ISSN 2347 – 517X

Original Article

INTERNATIONAL JOURNAL OF RESEARCHES IN BIOSCIENCES, AGRICULTURE AND TECHNOLOGY

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TIME-FRACTIONAL TWO-TEMPERATURE HYGROTHERMOELASTIC RESPONSE IN COMPOSITE HOLLOW CYLINDER INDUCED BY RAMP-TYPE HEATING

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Communicated : 23.01.2023	Revision : 28.02.2023 & 08.03.2023 Accepted : 03.04.2023	Published : 30.05.2023

ABSTRACT:

A new mathematical model of time-fractional two-temperature transient heat and moisture diffusion that obey Fourier and Fick's laws has been constructed in the context of the hygrothermoelastic theory. This paper provides a connection between the concepts of hygrothermoelasticity with fractional order, one-temperature and two-temperature in the unsteady diffusion process having a Dufour and Soret effect. The governing equations consist of linearly uncoupled partial differential equations applied to composite material hollow circular cylinders subjected to hygrothermal loading. The solution to the problem is first obtained in the Laplace transform domain. Furthermore, a complex inversion transform formula based on a Fourier expansion is used to get the numerical solutions of the coupled diffusion equations. The graph depicts the numerical results for the graphite fibre-reinforced epoxy resin composition of T300/5208.

Keywords: - Hygrothermoelastic, two-temperature theory, integral transform approach, fractional calculus, numerical results.

INTRODUCTION :

The coupling of humidity and temperature in the diffusion process has a Dufour effect and the Soret effect. The former is the effect of water concentration (humidity) on the thermal diffusion process, and the latter is the effect of temperature distribution on humidity diffusion. However, due to the equation's simplicity, most of the coupled equations are derived from the chemical potential, Fourier's heat conduction equation and Fick's diffusion equation are adopted. In most of the literature, for simplicity, it is assumed that moisture and temperature variations are confined within a small range. All material properties are independent of both the temperature and the moisture and are coupled. However, the elastic deformation due to the humidity and the temperature is within a linear elastic range. The effects of moisture and temperature on stresses and displacements in composite materials have risen in popularity.

Henry [1] incorporates a diffusion principle that considers the relationship between temperature and moisture. Hartranft and Sih [2] expanded on the phenomenological claims that contributed to coupled equations governing the simultaneous diffusion of moisture and heat. Although the coefficients were applied to the fundamental thermodynamic properties, both physical models resulted in the same differential equations. It was also noted that when the moisture diffusion coefficient was kept temperature-dependent for symmetric boundary conditions, that produced no bending [3], whereas when the skew-symmetric boundary condition was taken as bending occurred [4]. Sih et al. presented analytical solutions for the coupled heat and moisture diffusion problems in the case of a

http://doi.org/10.29369/ijrbat.2023.02.1.0044

e-ISSN 2347 – 517X
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plate [5-7], a strip [8], a spherical cavity [9] and a circular cavity [10]. One of the highly cited literature reviews on hygrothermoelasticity was considered by Sih et al. [11] in their book. Chang et al. found analytical solutions for hygrothermal stresses in a hollow cylinder [12], a solid cylinder [13], and a double-layer annular cylinder [14] subjected to hygrothermal heating using the decoupling approach. Sugano et al. achieved an analytical solution for the hygrothermal stresses in a hollow cylinder [15] and a functionally graded material plate [16] subjected to a non-axisymmetric hygrothermal environment by employing a similar technique. Chiba derived an analytical solution for one-way coupled transient heat and moisture in a double-layered plate [17] and applied it to the hygrothermoelastic problem of a functionally graded material whose physical properties vary along the thickness direction [18].

In the context of Carrera's Unified Formulation (CUF), Brischetto [19] analyzed the hygrothermal loading effects in the bending of multilayered composite plates, enabling classical models to be obtained. Ishihara [20] developed a system of nonlinear coupling diffusion equations for an infinite strip in porous media exposed to heat and moisture. The static behaviour of a functionally graded magnetoelectroelastic hollow sphere resting on a Winkler elastic foundation and subjected to hygrothermal stress was determined by Saadatfar and Aghaie-Khafri [21]. Zenkour [22] obtained the analytical solution to describe the hygrothermal responses in inhomogeneous piezoelectric hollow cylinders subjected to both mechanical load and electric potential. Zhao et al. [23] have used differential operator theory and the superposition principle to extract the general steady-state solution for three-dimensional hygrothermoelastic media with the potential theory's aid approach. Benkhedda et al. [24] developed an approximate model to estimate hygrothermoelastic stress in composite laminated plates during moisture desorption while accounting for changes in mechanical properties caused by temperature and moisture variations. Most of the previously published articles exposed composite structures' static and dynamic properties in the hygrothermal environment. Neither has looked at the theoretical context for coupling classical Fourier's and Fick's laws to establish a new two-temperature hygrothermoelastic diffusion principle for a non-simple rigid substance. Neither has looked at the theoretical context for coupling classical Fourier's and Fick's laws to establish a new two-temperature hygrothermoelastic diffusion principle for a non-simple rigid substance. The author, in their paper, proposed a system of linearly coupled partial differential equations for the thermal and moisture diffusion for the case of a non-simple medium [25].

Fractional or non-integer order calculus has recently been applied in physics, geology, chemistry, rheology, architecture, bioengineering, robotics, and other areas as a natural extension of classical differential and integral calculus. Many scientists have looked at fractional order differential equations as a way to explain anomalous diffusion in complex structures, like amorphous, porous, random, and disordered materials, fractal polymers, glasses, dielectrics and semiconductors, and so on [26]. Chaves [27] proposed a fractional-derivatives diffusion equation that generates the Lévy statistics based on a proposed generalization of Fick's law. Gorenflo et al. [28] obtained the time-fractional diffusion equation from the standard diffusion equation by replacing the first-order time derivative with a fractional derivative of order $\beta \in (0,1)$. Povstenko [29-32] proposed a series of academic papers on the time-fractional diffusion equation using fractional calculus methods, analogous to fractional Fick's principle. Very recently, Zhang

http://doi.org/10.29369/ijrbat.2023.02.1.0044

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[33-36] formulated a few coupled hygrothermoelasticity manuscripts within the fractional calculus framework to obtain closed-form expressions for temperature, moisture, and stresses. As far as the authors are aware, no analytical investigations that dealt with equations for the coupled model with the time-fractional two-temperature hygrothermal model were left open for further research. Similarly, even from the perspective of numerical analysis, the behaviour of the transient hygrothermal stresses in non-simple composite bodies has not yet been reported.

This article aims to describe the effects of time-fractional two-dimensional transient coupled heat and moisture diffusion on the elastic stresses in an infinitely long hollow cylinder under hygrothermal loadings. This article aims to describe the effects of time-fractional two-dimensional transient coupled heat and moisture diffusion on the elastic stresses in an infinitely long hollow cylinder under hygrothermal loadings. The classical Fick's law and Fourier heat conduction principle compare temperature and moisture fields, deformation, and stresses for various fractional orders.

Time-fractional two-temperature hygrothermoelastic theory

Henry [1] proposed an alternative microscopic approach as the first approximation for the variation of moisture and temperature as $M = \text{constant} + \sigma C - \omega T$, where σ and ω are material constants, C is the mass of moisture, and T is the temperature. Then the amount of moisture in composite per unit mass of solid, m, can be expressed as $m = \upsilon' C + \rho M$, with υ' as the volume fraction of the voids and ρ as the density of the material [2]. Due to the presence of liquid and vapour, the moisture and heat transfer obey the following conservation of mass and energy as $\nabla \cdot q_m = -\partial(m/\upsilon')/\partial t$ and $\nabla \cdot q_h = \partial(\rho\gamma M)/\partial t - \partial(\rho C_v T)/\partial t$.

Eliminating M, one obtains a system of simultaneous equations for moisture and thermal diffusion

$$\mathsf{D}\nabla^2 T = \frac{\partial T}{\partial t} - \upsilon_c \frac{\partial C}{\partial t} \tag{1}$$

$$D\nabla^2 C = \frac{\partial C}{\partial t} - \lambda_c \frac{\partial T}{\partial t} \tag{2}$$

where λ_c is the adiabatic coefficient, υ_c is an isothermal coefficient, D is the thermal diffusion coefficient under the state of constant vapour concentration, D is vapour diffusion coefficient under isothermal condition, and it is represented as [2,15]

$$\mathsf{D} = (1-\lambda_c\upsilon_c)D_h,\ D = (1-\lambda_c\upsilon_c)D_m \tag{3}$$

with D_h representing the coefficients of diffusion of heat and D_m stands for the diffusion of moisture, respectively.

Now, introducing the two temperatures models (2TT) are related by [37-43]

$$\phi = T - b\nabla^2 T,\ b > 0 \tag{4}$$

in which ϕ is the change in temperature or thermodynamic temperature, T is the conductive temperature, ∇^2 denotes the Laplace operator and b is the temperature discrepancy factor or parameter of the two-temperature model. In the limiting case, as $b \to 0,\ \phi \to T$ and the one-temperature models (1TT), are recovered.

The present problem in a non-simple medium can be written as

$$\mathsf{D}\left(1 + \frac{b}{\kappa}\frac{\partial}{\partial t}\right)\nabla^2 T = \frac{\partial T}{\partial t} - \upsilon_c \frac{\partial C}{\partial t} \tag{5}$$

$$D\nabla^2 C = \frac{\partial C}{\partial t} - \lambda_c \frac{\partial T}{\partial t} \tag{6}$$

in which thermal diffusivity is taken as $\kappa = \lambda / \rho C_v$, λ is the thermal conductivity of the material, ρ is the density, C_v is the calorific capacity, respectively.

http://doi.org/10.29369/ijrbat.2023.02.1.0044

e-ISSN 2347 – 517X
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Following [27-36], it is assumed that heat and moisture obey time-fractional Fourier and Fick's laws in which the matter flux has the power-law time-nonlocal kernel describing "long-tale" memory. Thus, the heat flux vector and moisture flux vector take the following form [33] as

$$q_h(t)=\begin{cases}-\dfrac{D}{\Gamma(\alpha)}\dfrac{\partial}{\partial t}\int_0^t (t-\tau)^{\alpha-1}\nabla T(\tau)\,d\tau, & 0<\alpha\le 1\\ -\dfrac{D}{\Gamma(\alpha-1)}\dfrac{\partial}{\partial t}\int_0^t (t-\tau)^{\alpha-2}\nabla T(\tau)\,d\tau, & 1<\alpha\le 2\end{cases}$$

$$q_m(t)=\begin{cases}-\dfrac{D}{\Gamma(\beta)}\dfrac{\partial}{\partial t}\int_0^t (t-\tau)^{\beta-1}\nabla C(\tau)\,d\tau, & 0<\beta\le 1\\ -\dfrac{D}{\Gamma(\beta-1)}\dfrac{\partial}{\partial t}\int_0^t (t-\tau)^{\beta-2}\nabla C(\tau)\,d\tau, & 1<\beta\le 2\end{cases}$$

in which α,β represents the fractional order of a Caputo fractional derivative with respect to time *t*, and $\Gamma(\ )$ is the Gamma function.

Thus, we get a system of linearly coupled partial differential equations as

$$D\left(1+\frac{b}{\kappa}\frac{\partial}{\partial t}\right)\nabla^2 T=\frac{\partial^\alpha T}{\partial t^\alpha}-\upsilon_c\frac{\partial^\alpha C}{\partial t^\alpha},0<\alpha\le 2$$

(7)

$$D\nabla^2 C=\frac{\partial^\beta C}{\partial t^\beta}-\lambda_c\frac{\partial^\beta T}{\partial t^\beta},0<\beta\le 2$$

(8)

in which $(\partial^\alpha/\partial t^\alpha)$ and $(\partial^\beta/\partial t^\beta)$ is the Caputo fractional derivative, and Caputo fractional derivative is written as [44]

$$\frac{d^\alpha f(t)}{dt^\alpha}=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\dfrac{d^n f(\tau)}{d\tau^n}d\tau, & n-1<\alpha<n\\ \dfrac{d^n f(\tau)}{dt^n}, & \alpha=n\end{cases}$$

(9)

As a particular case of hygrothermoelasticity theory,

(i) Taking $b=0,\ \alpha=1$ and $\beta=1$ in Eqs. (7)-(8), the equations of the one-temperature model (1TT) can be found as obtained by Sugano and Chuuman [15].

(ii) Taking $b=0$ in Eqs. (7)-(8), the equations for the coupled model with different fractional orders can be obtained as suggested by Zhang and Li [33].

(iii) Taking $b=0$ and $\alpha=\beta$ in Eqs. (7)-(8), the equations for the coupled model with equal fractional orders can be achieved as proposed by Zhang and Li [33].

Formulation of the problem

Consider an infinitely long hollow circular cylinder with an inner radius $r=r_a$ and outer radius $r=r_b$, which is subjected to nonaxisymmetric hygrothermal loading, that is, (T_a,C_a) at the curved inner surface and (T_b,C_b) at the curved outer surface, as shown in Figure 1.

The system of linearly coupled equations for the composite material hollow circular cylinder can be taken as Eqs. (7) and (8), subject to boundary conditions

$$T|_{t=0}=T_i,C|_{t=0}=C_i,\left.\frac{\partial T}{\partial t}\right|_{t=0}=\left.\frac{\partial C}{\partial t}\right|_{t=0}=0,T|_{\gamma=0}=C|_{\gamma=0}=0,$$
$$T|_{r=r_a}=T_a f_a(\theta),C|_{r=r_a}=C_a g_a(\theta),T|_{r=r_b}=T_b f_b(\theta),C|_{r=r_b}=C_b g_b(\theta)$$

(10)

where $T(r,\theta,t)$ and $C(r,\theta,t)$ are the temperature and moisture, T_i and C_i are the reference temperature and moisture at the initial state, and

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

The components of strain associated with plane-strain, compatibility conditions of strain, equilibrium equations, stress-strain components disregarding the body forces $(\text{i.e.} F_r=F_\theta=0)$, and components of stress are given by

http://doi.org/10.29369/ijrbat.2023.02.1.0044

e-ISSN 2347 – 517X

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$$\varepsilon_{rr}=\frac{1-\upsilon^2}{E}\left(\sigma_{rr}-\frac{\upsilon}{1-\upsilon}\sigma_{\theta\theta}\right)+\alpha_t(1+\upsilon)T+\beta_t(1+\upsilon)C,$$

$$\varepsilon_{\theta\theta}=\frac{1-\upsilon^2}{E}\left(\sigma_{\theta\theta}-\frac{\upsilon}{1-\upsilon}\sigma_{rr}\right)+\alpha_t(1+\upsilon)T+\beta_t(1+\upsilon)C,$$

$$\varepsilon_{\theta\theta}=\frac{1+\upsilon}{E}\sigma_{r\theta},$$

(11)

$$\frac{\partial^2\varepsilon_{rr}}{\partial\theta^2}+r^2\frac{\partial^2\varepsilon_{\theta\theta}}{\partial r^2}+2r\frac{\partial\varepsilon_{\theta\theta}}{\partial r}-r\frac{\partial\varepsilon_{rr}}{\partial r}-2\frac{\partial^2(r\varepsilon_{r\theta})}{\partial r\,\partial\theta}=0,$$

(12)

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0,\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2\sigma_{r\theta}}{r}=0,$$

(13)

$$\sigma_{rr}=\frac{1}{r^2}\frac{\partial^2\chi}{\partial\theta^2}+\frac{1}{r}\frac{\partial\chi}{\partial r},\sigma_{\theta\theta}=\frac{\partial^2\chi}{\partial r^2},\sigma_{r\theta}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\chi}{\partial\theta}\right).$$

(14)

It is easily found that the equilibrium equations (13) are automatically satisfied by the introduction of the thermal stress function $\chi(r,\theta,t)$. Since the solution of the problem should satisfy the stress-strain compatibility equation (11), substitution of components of stress (14) into the compatibility equation (12) and using Eq. (11), yields

$$\nabla^2\nabla^2\chi=-\frac{E}{1-\upsilon}(\alpha_t\nabla^2T+\beta_t\nabla^2C).$$

(15)

The set of equations (4) to (15) constitutes the mathematical hygrothermoelastic formulation within a hollow circular cylinder at any instant.

Solution

Introducing the following dimensionless parameters for ease of analysis

$$r'=r/r_a,t'=(\mathrm{D}/r_a{}^2)^{1/\alpha}t,b'=(\mathrm{D}/r_a{}^2)^{1/\alpha}b,\ \phi'=(\phi-T_i)/T_i,$$

$$T'=(T-T_i)/T_i,C'=(C-C_i)/\lambda_cC_i,K=[(D/r_a{}^2)^{1/\beta}(r_a{}^2/\mathrm{D})^{1/\alpha}]^\beta$$

(16)

Using these non-dimensional variables, Eqs. (7), (8) and (10) take the form

$$K\nabla'^2C'=\frac{\partial^\beta C'}{\partial t'^\beta}-\frac{\partial^\beta T'}{\partial t'^\beta} \quad (17)$$

$$\left(1+\frac{b'}{\kappa}\frac{\partial}{\partial t'}\right)\nabla'^2T'=\frac{\partial^\alpha T'}{\partial t'^\alpha}-\upsilon_c\frac{\partial^\alpha C'}{\partial t'^\alpha} \quad (18)$$

subject to boundary conditions

$$T'|_{t'=0}=C'|_{t'=0}=\left.\frac{\partial T'}{\partial t'}\right|_{t'=0}=\left.\frac{\partial C'}{\partial t'}\right|_{t'=0}=0,T'|_{\gamma=0}=C'|_{\gamma=0}=0,$$

$$T'|_{r'=1}=T'_af_a(\theta),C'|_{r'=1}=C'_ag_a(\theta),T'|_{r'=\xi}=T'_bf_b(\theta),C'|_{r'=\xi}=C'_bg_b(\theta)$$

(19)

where the primes stand for dimensionless quantities, dimensionless radius as $\xi=r_b/r_a$ and

$$\nabla'^2=\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}+\frac{1}{r'^2}\frac{\partial^2}{\partial\theta^2}.$$

For convenience, the primes will be dropped from here on. It was here dropping the primes for convenience.

The following property of the Laplace transform of the Caputo derivative operator [44] can be used to derive the solutions of Eqs. (17)-(19)

$$\mathrm{L}\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\}=p^\alpha f^*(p)-\sum_{m=0}^{n-1}p^{\alpha-1-m}\frac{\partial^m f(0)}{\partial t^m},n-1<\alpha<m,$$

(20)

where p is the Laplace transform parameter and f^* is the Laplace transform of f.

Applying Laplace transform [45] defined as $\bar{f}(p)=\int_0^\infty\exp[-pt]f(t)dt$ to both sides of Eqs. (17) and(18), and taking the initial conditions (19), one obtains

$$K\nabla^2C^*=p^\beta(C^*-T^*), \quad (21)$$

$$\left(1+\frac{b}{\kappa}p\right)\nabla^2T^*=p^\alpha(T^*-\upsilon_cC^*), \quad (22)$$

$$T^*|_{\gamma=0}=C^*|_{\gamma=0}=0,$$

$$T^*|_{r=1}=T_af_a(\theta),C^*|_{r=1}=C_ag_a(\theta),T^*|_{r=\xi}=T_bf_b(\theta),C^*|_{r=\xi}=C_bg_b(\theta)$$

(23)

Applying the finite Fourier sine transform [45] defined as $\bar{f}(\nu)=\int_0^\gamma f(\theta)\sin(\nu\pi\theta/\gamma)d\theta$ to both sides of Eqs. (21) and (22), and taking the boundary conditions (23), one yield

$$K\bar{\nabla}^2\bar{C}^*=p^\beta(\bar{C}^*-\bar{T}^*), \quad (24)$$

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$$\left(1+\frac{b}{\kappa}p\right)\bar{\nabla}^2\bar{T}^* = p^{\alpha}(\bar{T}^* - \upsilon_c \bar{C}^*) \quad ,$$

(25)

$$\bar{T}^*\Big|_{r=1} = T_a \bar{f}_a(\theta), \bar{C}^*\Big|_{r=1} = C_a \bar{g}_a(\theta), \bar{T}^*\Big|_{r=\xi} = T_b \bar{f}_b(\theta), \bar{C}^*\Big|_{r=\xi} = C_b \bar{g}_b(\theta),$$

(26)

$$\bar{\nabla}^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\nu^2 \pi^2}{\gamma^2}$$

(27)

Introducing the finite Hankel integral transform [45] and its inversion theorem as

$$\bar{f}_n(\mu_i) = \int_a^b r\, f(r)\, B_n(r\mu_i)\, dr,\ b > a,$$

$$f(r) = \frac{\pi^2}{2}\sum_n \frac{\mu_i^2 \bar{f}_n(\mu_i) B_n(r\mu_i) J_n^2(b\mu_i)}{J_n^2(a\mu_i) - J_n^2(b\mu_i)}$$

(28)

where n be the transform parameter and kernel for the finite transform defined by

$$B_n(r\mu_i) = J_n(r\mu_i)Y_n(a\mu_i) - Y_n(r\mu_i)J_n(a\mu_i)$$

(29)

with μ_i are the positive roots of the characteristic equation $B_n(b\mu_i) = 0$ and $Y_n(x)$ is the Bessel function of the second kind of order n.

Now perform the finite Hankel transform to Eq. (24) and (25), using the boundary conditions (26), leading to

$$K(-\mu_i^2 \bar{\bar{C}}^* + A_2) = p^{\beta}(\bar{\bar{C}}^* - \bar{\bar{T}}^*),$$

(30)

$$A_3(-\mu_i^2 \bar{\bar{T}}^* + A_1) = p^{\alpha}(\bar{\bar{T}}^* - \upsilon_c \bar{\bar{C}}^*),$$

(31)

where

$$A_1 = \frac{2}{\pi}\left[T_b \bar{f}_b(\theta)\left(\frac{J_n(a\mu_i)}{J_n(b\mu_i)}\right) - T_a \bar{f}_a(\theta)\right],$$

$$A_2 = \frac{2}{\pi}\left[C_b \bar{g}_b(\theta)\left(\frac{J_n(a\mu_i)}{J_n(b\mu_i)}\right) - C_a \bar{g}_a(\theta)\right],$$

$$A_3 = 1 + \frac{b}{\kappa}p,\ n = \pi\nu/\gamma.$$

Now eliminating $\bar{\bar{T}}^*$ from the Eqs. (30) and (31), one obtains

$$\bar{\bar{C}}^* = \frac{KA_2(\mu_i^2 A_3 + p^{\alpha}) + p^{\beta} A_3 A_1}{(K\mu_i^2 + p^{\beta})(\mu_i^2 A_3 + p^{\alpha}) - \upsilon_c p^{\alpha} p^{\beta}} \quad .$$

(32)

Substituting Eq. (32) into Eq. (30), one gets

$$\bar{\bar{T}}^* = \frac{(K\mu_i^2 + p^{\beta})}{p^{\beta}}\left[\frac{KA_2(\mu_i^2 A_3 + p^{\alpha}) + p^{\beta} A_3 A_1}{(K\mu_i^2 + p^{\beta})(\mu_i^2 A_3 + p^{\alpha}) - \upsilon_c p^{\alpha} p^{\beta}}\right] - \frac{KA_2}{p^{\beta}}$$

. (33)

Applying the inversion theorems of transformation on Eqs.(29) and (33), and obtaining the moisture and temperature change in the Laplace domain below

$$C^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} A_4 \left\{\frac{KA_2(\mu_i^2 A_3 + p^{\alpha}) + p^{\beta} A_3 A_1}{(K\mu_i^2 + p^{\beta})(\mu_i^2 A_3 + p^{\alpha}) - \upsilon_c p^{\alpha} p^{\beta}}\right\},$$

(34)

$$T^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} \frac{A_4}{p^{\beta}}\left\{(K\mu_i^2 + p^{\beta})\left[\frac{KA_2(\mu_i^2 A_3 + p^{\alpha}) + p^{\beta} A_3 A_1}{(K\mu_i^2 + p^{\beta})(\mu_i^2 A_3 + p^{\alpha}) - \upsilon_c p^{\alpha} p^{\beta}}\right] - KA_2\right\},$$

(35)

where

$$A_4 = \frac{\pi \mu_i^2 B_n(r\mu_i) J_n^2(b\mu_i)}{J_n^2(a\mu_i) - J_n^2(b\mu_i)}\sin(n\theta).$$

We now introduce the non-dimensional variable

$$\chi' = (1-\upsilon)\chi / E r_a T_i \quad ,$$

$$\sigma'_{ij} = (1-\upsilon)\sigma_{ij} / E\alpha_t T_i \ (i, j = r, \theta)$$

(36)

in dimensionless form for ease of analysis (for convenience, we drop the primes from here on) and taking the Laplace integral transforms on both sides of Eqs. (14) and (15) as follows

$$\sigma_{rr}^* = \frac{1}{r^2}\frac{\partial^2 \chi^*}{\partial \theta^2} + \frac{1}{r}\frac{\partial \chi^*}{\partial r}, \sigma_{\theta\theta}^* = \frac{\partial^2 \chi^*}{\partial r^2}, \sigma_{r\theta}^* = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \chi^*}{\partial \theta}\right).$$

(37)

$$\nabla^2\nabla^2 \chi^* = -(\alpha_t \nabla^2 T^* + \beta\lambda_c \nabla^2 C^*),$$

(38)

Assuming stress function χ^* which satisfies Eq. (38) as

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$$\chi^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} \frac{A_4}{p^\beta}\mu_i^2 \left\{ [\alpha_t (K\mu_i^2 + p^\beta) + \beta_t \lambda_c p^\beta] \times \left[\frac{KA_2(\mu_i^2 A_3 + p^\alpha) + p^\beta A_3 A_1}{(K\mu_i^2 + p^\beta)(\mu_i^2 A_3 + p^\alpha) - \upsilon_c p^\alpha p^\beta} \right] - \alpha_t K A_2 \right\}$$

(39)

Now using Eqs. (39) in (14), one obtains the expressions for stresses as

$$\sigma_{rr}^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} \frac{A_5}{p^\beta}\mu_i^2 \left\{ [\alpha_t (K\mu_i^2 + p^\beta) + \beta_t \lambda_c p^\beta] \times \left[\frac{KA_2(\mu_i^2 A_3 + p^\alpha) + p^\beta A_3 A_1}{(K\mu_i^2 + p^\beta)(\mu_i^2 A_3 + p^\alpha) - \upsilon_c p^\alpha p^\beta} \right] - \alpha_t K A_2 \right\},$$

(40)

$$\sigma_{\theta\theta}^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} \frac{A_6}{p^\beta}\mu_i^2 \left\{ [\alpha_t (K\mu_i^2 + p^\beta) + \beta_t \lambda_c p^\beta] \times \left[\frac{KA_2(\mu_i^2 A_3 + p^\alpha) + p^\beta A_3 A_1}{(K\mu_i^2 + p^\beta)(\mu_i^2 A_3 + p^\alpha) - \upsilon_c p^\alpha p^\beta} \right] - \alpha_t K A_2 \right\},$$

(41)

$$\sigma_{r\theta}^* = \sum_{\nu=1}^{\infty}\sum_{i=1}^{\infty} \frac{A_7}{p^\beta}\mu_i^2 \left\{ [\alpha_t (K\mu_i^2 + p^\beta) + \beta_t \lambda_c p^\beta] \times \left[\frac{KA_2(\mu_i^2 A_3 + p^\alpha) + p^\beta A_3 A_1}{(K\mu_i^2 + p^\beta)(\mu_i^2 A_3 + p^\alpha) - \upsilon_c p^\alpha p^\beta} \right] - \alpha_t K A_2 \right\},$$

(42)

where

$$A_5 = \frac{\pi \mu_i^2 [J_n'(r\mu_i)Y_n(a\mu_i) - Y_n'(r\mu_i)J_n(a\mu_i)] J_n^2(b\mu_i)}{J_n^2(a\mu_i) - J_n^2(b\mu_i)} \sin(n\theta)$$

,

$$A_6 = \frac{\pi \mu_i^2 [J_n''(r\mu_i)Y_n(a\mu_i) - Y_n''(r\mu_i)J_n(a\mu_i)] J_n^2(b\mu_i)}{J_n^2(a\mu_i) - J_n^2(b\mu_i)} \sin(n\theta)$$

,

$$A_7 = \frac{\pi n \mu_i^2 [J_n'(r\mu_i)Y_n(a\mu_i) - Y_n'(r\mu_i)J_n(a\mu_i)] J_n^2(b\mu_i)}{J_n^2(a\mu_i) - J_n^2(b\mu_i)} \cos(n\theta)$$

.

The numerical inversion of the Laplace transforms

Eqs. (34), (35), and (40)-(42) provide the expressions for moisture, temperature and hygrothermal stress in the Laplace domain. It is difficult to find the analytical inverse Laplace's transform of the complicated solutions in Laplace's transform domain. In order to determine these in the physical domain, we adopt a numerical inversion method based on a Fourier series expansion of functions performed by Honig and Hirdes [46]. In this method, the inverse $f(t)$ of the Laplace transform $f(s)$ is approximated by the relation

$$f(t) = \frac{e^{st}}{t_1}\left[\frac{1}{2}\bar{f}(s) + \mathrm{Re}\sum_{k=1}^{N} \bar{f}\left(s + \frac{ik\pi}{t_1} \right) \exp\left(\frac{ik\pi}{t_1} \right) \right], 0 \le t_1 \le t$$

(43)

where N is a sufficiently larger integer representing the number of terms in the truncated Fourier series, chosen such that

$$f(t) = e^{st}\,\mathrm{Re}\left[\bar{f}\left(s + \frac{iN\pi}{t_1} \right) \exp\left(\frac{iN\pi t}{t_1} \right) \right] \le \varepsilon_1,$$

in which ε_1 is a prescribed small positive value that corresponds to the degree of accuracy to be achieved.

Numerical Results, Discussion and Remarks

We consider the composite hollow cylinder was prepared from Thornel (Union Carbide) T300 graphite fibres and Narmco 5208 epoxy resin. The graphite fiber-reinforced epoxy matrix composite (T300/5208) was chosen for numerical calculations with the following material properties.

$$\alpha_t = 31.3\times 10^{-6}\ \mathrm{cm/cm\times K},\ \beta_t = 2.68\times 10^{-3}\ \mathrm{cm/cm\times K\ wt\,\%H_2O},$$
$$E = 64.30\,\mathrm{GPa},\ \upsilon = 0.33,\ \kappa = 0.216\ \mathrm{cm^2/s}, C_\infty = 1.55\%\ \mathrm{dry\ wt},$$
$$D_h = 6.90\times 10^{-6}\ \mathrm{cm^2/hr},\ D_m = 6.90\times 10^{-7}\ \mathrm{cm^2/hr},$$

(44)

The physical parameters as

$$T_i = 293.16\,\mathrm{K},\ T_a = 373.16\,\mathrm{K},\ T_b = 293.16\,\mathrm{K},$$
$$C_i = 0.0\%\ \mathrm{dry\ wt},\ C_a = 1.55\%\ \mathrm{dry\ wt},\ C_b = 0.0\%\ \mathrm{dry\ wt},$$
$$\gamma_{f(g)} = 0.7,\ \gamma_g = 0 \square 1.0,\ \lambda_c = 0.122\,\mathrm{kg/m^3K},$$
$$\upsilon_c = 2.053\,\mathrm{m^3K/kg},\ \alpha_c = 90^\circ,\ \Delta\alpha_c = 15^\circ,\ RH = 70\%$$

(45)

and the prescribed surface temperature as

$$f_a(\theta), g_a(\theta) = \begin{cases} 1, & 0 \le \theta \le \theta_1 \\ f_\psi(\theta), & \theta_1 \le \theta \le \theta_2 \\ \gamma_{f(g)}, & \theta_2 \le \theta \le \pi \end{cases}$$

(46)

and

http://doi.org/10.29369/ijrbat.2023.02.1.0044

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$$f_b(\theta) = g_b(\theta) = 1$$
(47)

where

$$f_\psi(\theta) = \left(\frac{1+\gamma_{f(g)}}{2}\right) + \left(\frac{1-\gamma_{f(g)}}{2}\right)\cos\left[\frac{\pi-[\theta-\theta_1]}{2\Delta\alpha_c}\right]$$

and

$$\theta_1 = \alpha_c - \Delta\alpha_c, \theta_2 = \alpha_c + \Delta\alpha_c$$

In order to examine the influence of hygrothermoelastic response in a composite hollow cylinder, numerical calculations were performed for all the variables, and numerical calculations are depicted in the following figures with the help of MATHEMATICA software.

CONCLUSION :

The proposed closed-form

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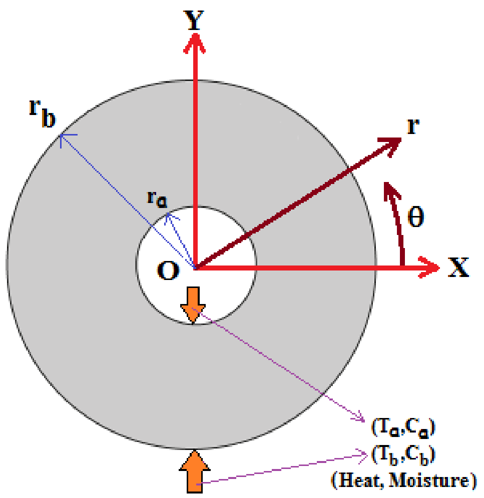

Figure 1. The geometry of a hollow circular cylinder

http://doi.org/10.29369/ijrbat.2023.02.1.0044